Mobile Robot
Kinematic Structure for Control Issues
Lecture 2

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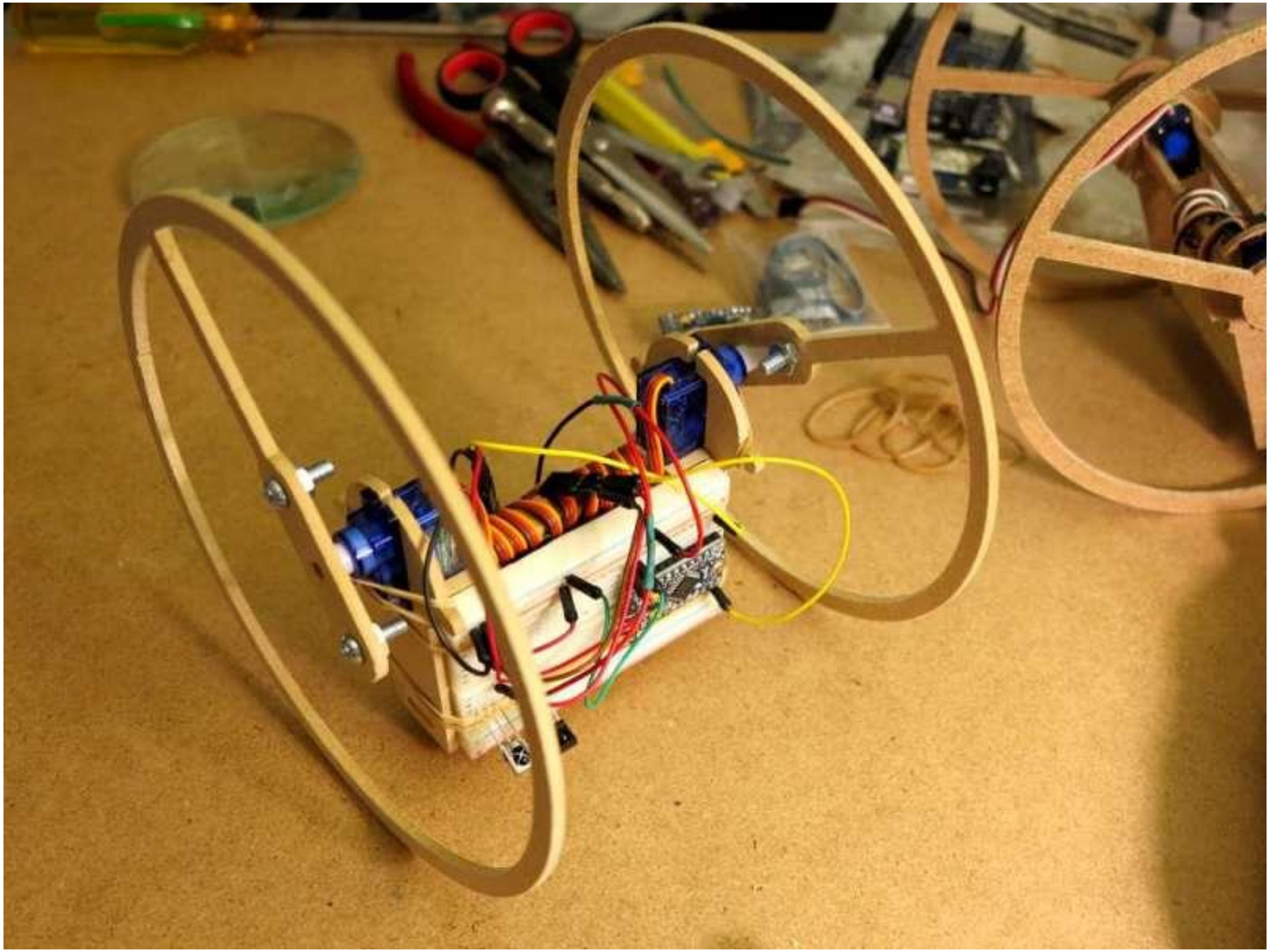
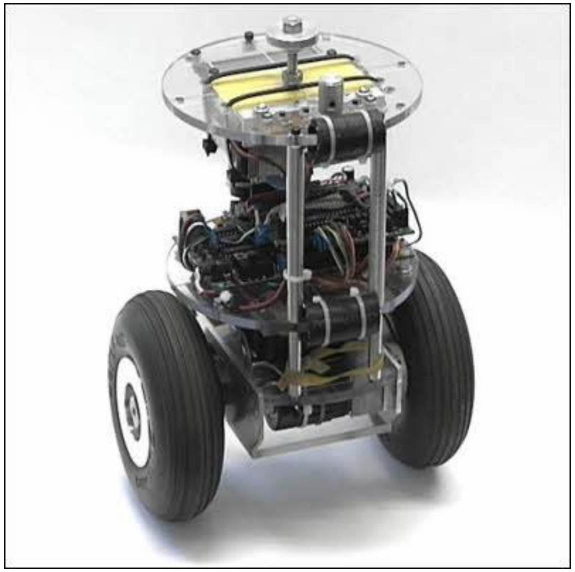
Wheel from Stone Age

- Maya did NOT use wheels.
  - Even pulley and tools.

Two Wheeled Robot



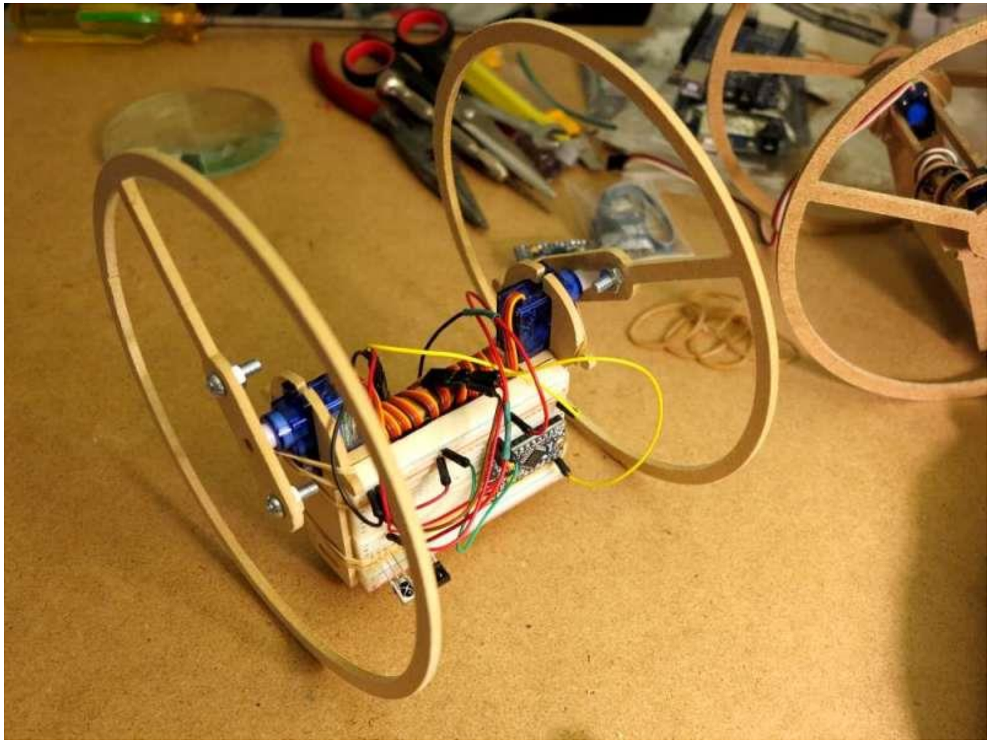
Two wheeled robot has some problems



- Falling down…
- Why it is unstable?

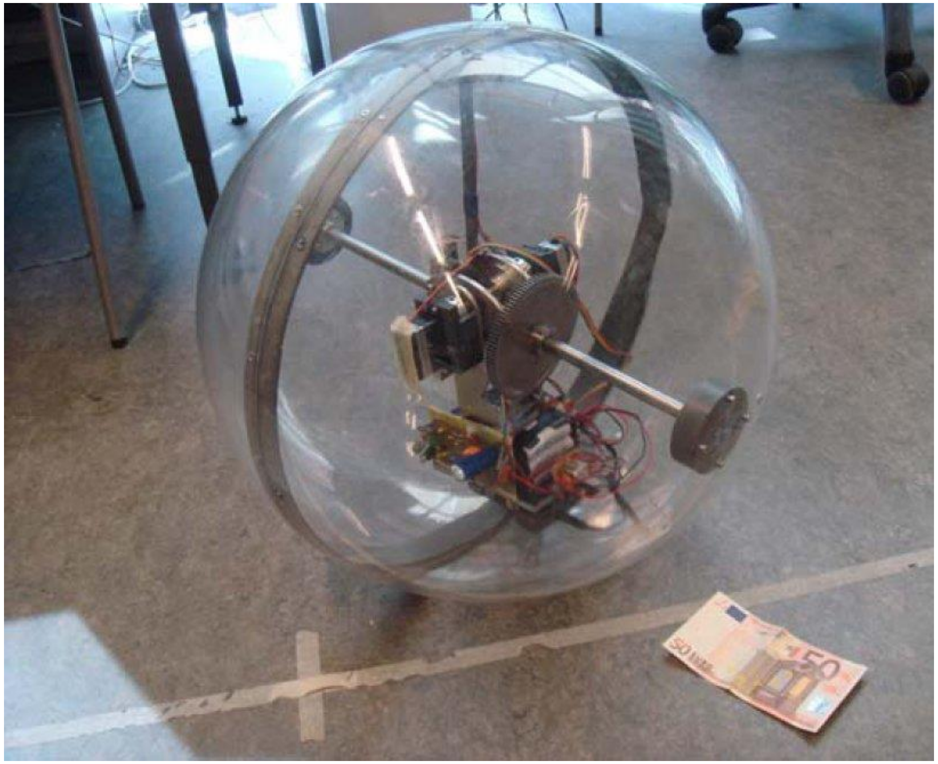
Stability Problem with wheels

Two wheeled robot has some problems



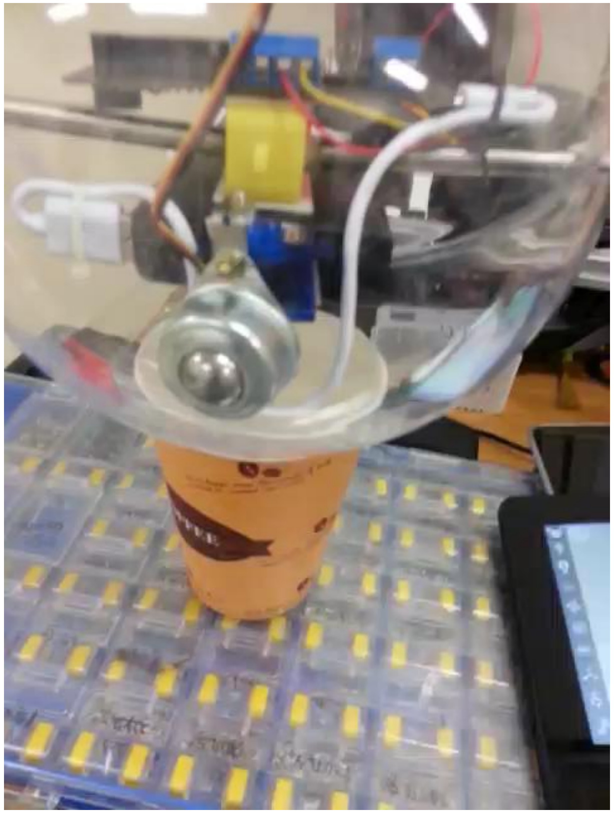
- It is stable, but angular position control has some noise.
- Why?

Two Wheeled Robot?
No, One wheel → Spherical Robot



- Balance control is required.
- But, very fast and low power consumption

Spherical Robot



- Tilt feedback is required.

For Mechanics Analysis

- F is derived from Linear Momentum

$$\lim_{\Delta t \to 0} \frac{\Delta L}{\Delta t} = \lim_{\Delta t \to 0} \frac{m\Delta v}{\Delta t} = ma = F$$

- Moment, M is derived from Angular Momentum
  - H= r x L = r x mv (x is a cross product)

$$\lim_{\Delta t \to 0} \frac{\Delta H}{\Delta t} = \lim_{\Delta t \to 0} r \times \frac{m\Delta v}{\Delta t} = r \times ma = r \times F = M\,(or\ =T)$$

- Moment is often called Torque.

Statics Equilibrium

- Force equilibrium

dynamics

$$\sum F = ma = 0 \qquad \sum F = ma \neq 0$$

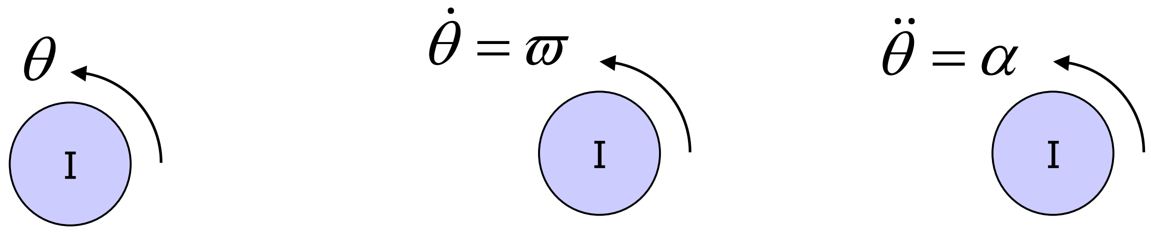
  - Sum of all external forces should be zero
  - If all force sum is zero, there is no movement by an acceleration

- Moment equilibrium

$$\sum M = r \times F = 0 \qquad \sum M = r \times F = I\alpha$$

  - Sum of all external moments should be zero
  - If moment is zero, there is NO rotation.

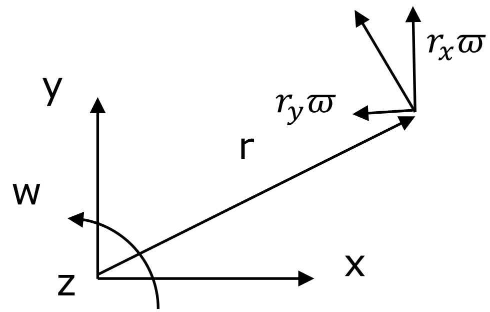
Angular Velocity and Acceleration



- Angle $\theta = \theta(t) \quad \leftarrow x = x(t)$
- Angular velocity $\varpi = \dot{\theta} = \dfrac{d\theta(t)}{dt} \quad \leftarrow v = \dot{x}$
- Angular Acceleration $\alpha = \ddot{\theta} = \dfrac{d^2\theta(t)}{dt^2} \quad \leftarrow a = \ddot{x}$

V = w x r

- V = $\dfrac{dr}{dt}$ =w x r (Cross product)



$$= \begin{vmatrix} i & j & k \\ w_x & w_y & w_z \\ r_x & r_y & r_z \end{vmatrix} = \begin{vmatrix} i & j & k \\ 0 & 0 & \varpi \\ r_x & r_y & 0 \end{vmatrix} = r_x\varpi j - r_y\varpi i$$

- A = $\dfrac{d^2r}{dt^2} = \alpha \times r$

Remind

$$\hat{x} = r\hat{e}_r$$

$$\dot{x} = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\ddot{x} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(2\dot{r}\dot{\theta} + r\ddot{\theta}\right)\hat{e}_\theta$$

$$\hat{x} = r\hat{e}_r \quad \text{r=const.}$$

$$\dot{x} = 0 + r\dot{\theta}\hat{e}_\theta$$

$$\ddot{x} = \left(0 + r\ddot{\theta}\right)\hat{e}_\theta$$

Moment of Inertia, I

- What is Momentum?
  - Conservation of Momentum: L = mv = const.
  - Time Differentiation of L
  - $$\dot{L} = \frac{dL}{dt} = \frac{dmv}{dt} = m\frac{dv}{dt} = ma$$
  - Angular Momentum, H = ?

$$H = r \times L = r \times mv = m\left(r \times v\right) = m\left(r \times \left(w \times r\right)\right)$$

  - Time Differentiation of H=?

$$\dot{H} = \frac{d}{dt}\left(r \times L\right) = \frac{d}{dt}\left(r \times mv\right) = \mathrm{m}\left[\frac{dr}{dt} \times v + r \times \frac{dv}{dt}\right]$$

Too complex.. T_T

Angular Moment of Inertia (for Rigid Body)

- Remind

$$\dot{H} = r \times \dot{L} = \frac{d}{dt}(r \times mv) = \mathrm{m}\left[\frac{dr}{dt} \times v + r \times \frac{dv}{dt}\right]$$

- Because it is a Rigid body,
  - r is a constant value. $\dot{r} = 0$

- Simplification

$$\dot{H} = r \times \dot{L} = \frac{d}{dt}(r \times mv) = r \times m\frac{dv}{dt} = mr \times \left(r \times \frac{dw}{dt}\right) = mr \times (r \times \alpha)$$

$$\longrightarrow \dot{H} = \sum_i m_i r_i \times (r_i \times \alpha)$$

Torque on Rigid body.
Remind that all α_i is α

Angular Momentum of Inertia

- Rotation of Particles on Origin.

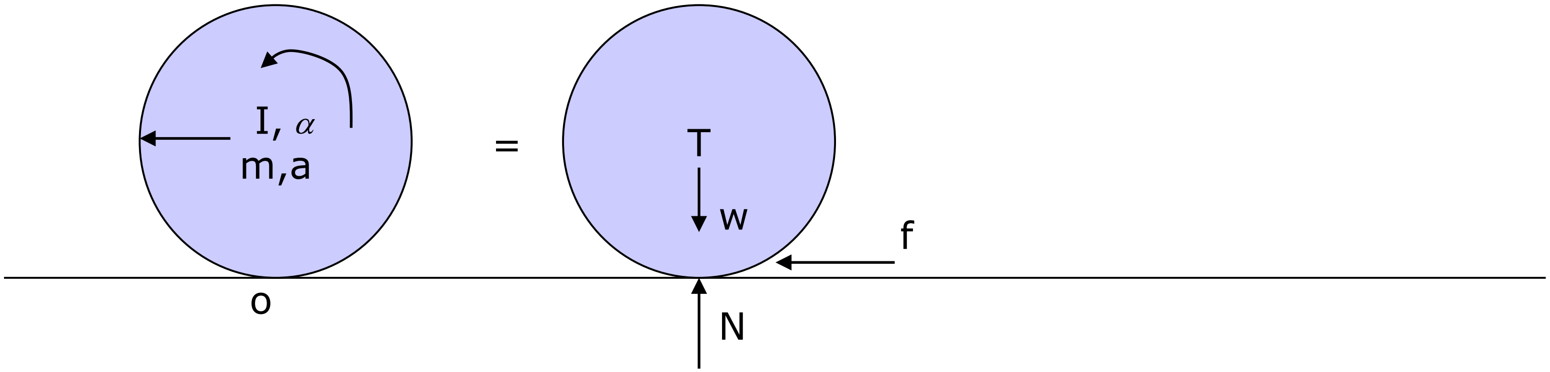
$$\dot{H} = \sum_i m_i r_i \times (r_i \times \alpha)$$

$$r_i \times (r_i \times \alpha) = r_i \times \begin{vmatrix} e_r & e_\theta & k \\ r_i & 0 & 0 \\ 0 & 0 & \alpha \end{vmatrix} = r_i \times (r_i\alpha)\hat{k} = \begin{vmatrix} e_r & e_\theta & k \\ r_i & 0 & 0 \\ 0 & 0 & r_i\alpha \end{vmatrix} = r_i^2\alpha$$

$$\dot{H} = \alpha \sum_i m_i r_i^2 = \alpha \int_m r^2 dm = I\alpha$$

$$\therefore I = \sum_i m_i r_i^2 = \int_m r^2 dm$$

Wheel Dynamics



$$1)\ ma_x = f$$

$$2)\ ma_y = 0 = -W + N$$

$$3)\ I_c\alpha = \left(\frac{1}{2}mr^2\right)\alpha = T - rf$$

$$\therefore \frac{1}{2}mr^2\alpha = T - rf$$

$$4)\ a_x = r\alpha$$

$$mr\alpha = f$$

$$\frac{1}{2}mr^2\alpha = T - mr^2\alpha$$

$$\therefore \alpha = T\frac{2}{3mr^2}$$

When T=const., if r ↓ then α ↑ but w ↓


vs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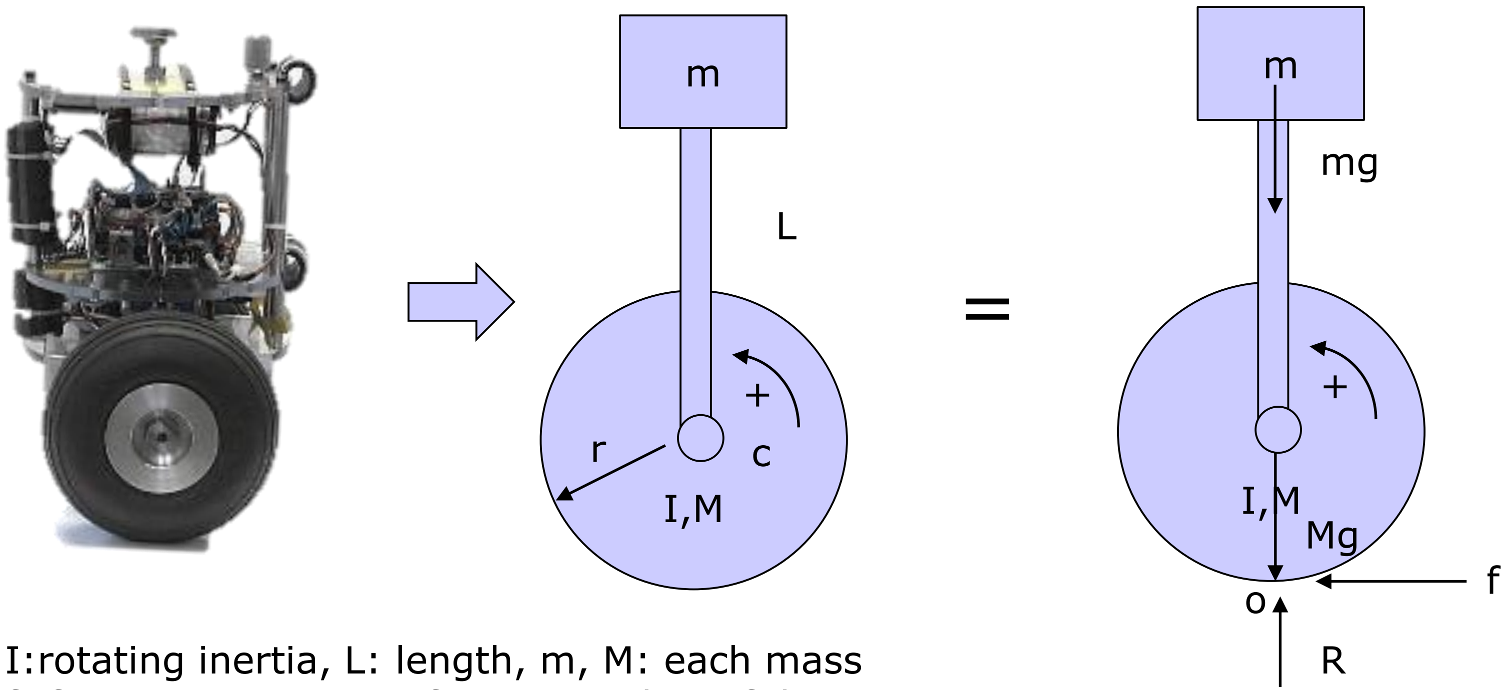
Dynamic Model of Two wheeled Robot



I:rotating inertia, L: length, m, M: each mass
f: friction, R: reaction force, r: radius of disc

$$\sum F_x = ma_x \qquad Ma + ma' = Mr\alpha + m(-r\alpha - L\alpha) = f$$

$$\sum F_y = ma_y \qquad -mg - Mg + R = 0$$

$$\sum M_c = r \times F = I\alpha \qquad I_c\alpha - mL^2\alpha = -rf$$

System Dynamics: Two wheeled Robot

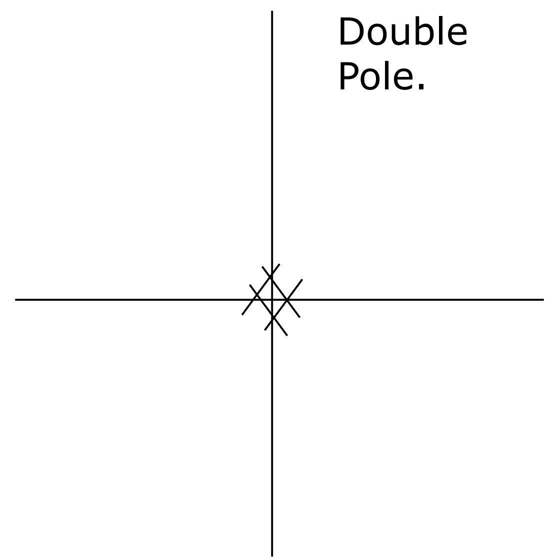
- If we cancel f with eq1,

$$\left(I_c - mL^2 + Mr^2 - mr(r+L)\right)\alpha = 0$$

$\therefore I^*\alpha = 0$ → Zero because there is no Torque Input

$\therefore I^*\ddot{\theta} = T$ → Add Torque Input

Double
Pole.



Laplace Transform

$$\frac{\Theta(s)}{\mathrm{T}(s)} = G(s) = \frac{1}{I^* s^2}$$

$= \textit{system}$ dynamics with Two Poles

Remind that
Poles on jw axis mean
Marginal stable

What is Double Pole? Remind mx''=F

Double Pole has No Damping

$$m\ddot{x} = F$$

Given system even without Damping and Feedback control

$$e = x_d - x$$

$$F = K_p e + K_d \dot{e}$$

PD Feedback Control

$$m\ddot{x} = K_p e + K_d \dot{e} = K_p (x_d - x) + K_d (\dot{x}_d - \dot{x})$$

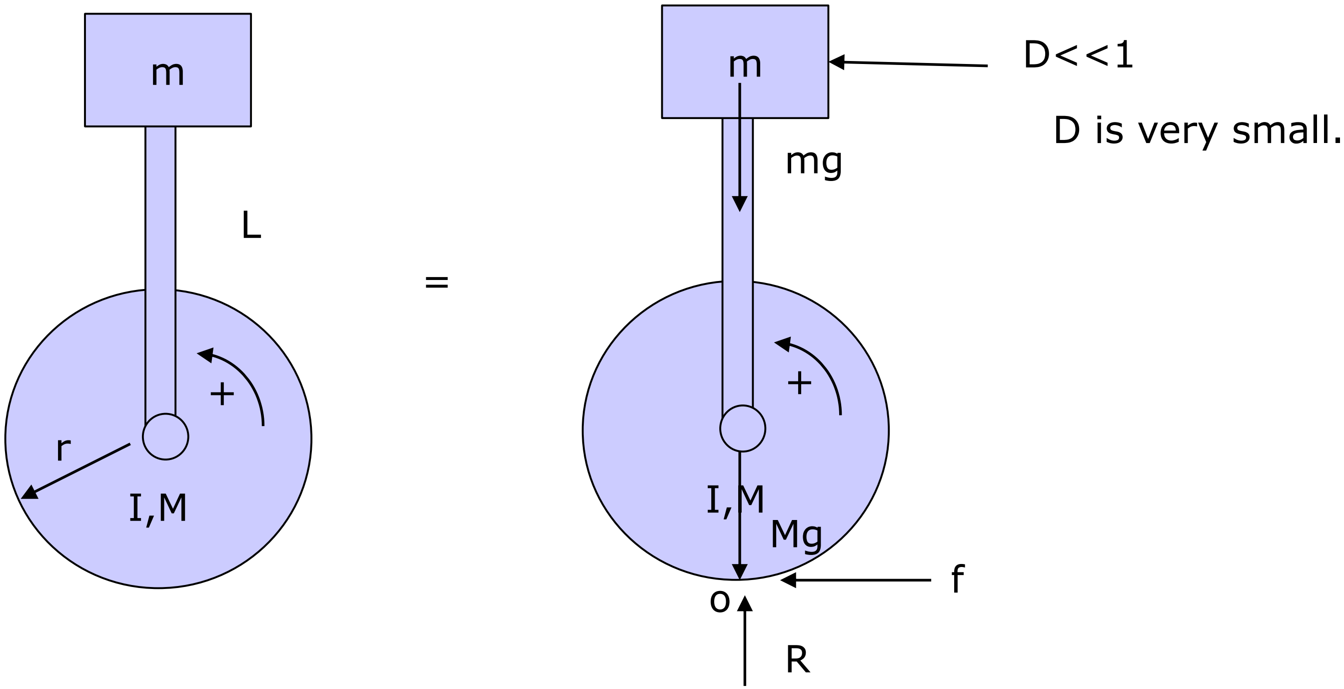
$$m\ddot{x} + K_d \dot{x} + K_p x = K_p x_d$$

$$\therefore m\ddot{x} + c\dot{x} + kx = F = K_p x_d$$

D control changes system dynamics (Stiffness, Damping)

→Remind that PD does not change on inertia $m\ddot{x}$

Disturbances on Marginal Stable System



$$\sum F_x = ma_x \qquad Ma + ma' = Mr\alpha - m(r+L)\alpha = -f - D$$

$$\sum M_o = r \times F = I\alpha \qquad I_c\alpha - mL^2\alpha = -rf + DL$$

$$\sum F_x = ma_x \qquad Ma + ma' = Mr\alpha - m(r+L)\alpha = -f - D$$

$$I_c\alpha - mL^2\alpha = -rf + DL$$ Without no Torque Input

$$I * \alpha = (L+r)D$$

Stability of Torque Input

$$I^*\ddot{\theta} = T$$

$$\frac{\Theta(s)}{\mathrm{T}(s)} = G(s) = \frac{1}{I^* s^2}$$

Stability of Disturbances

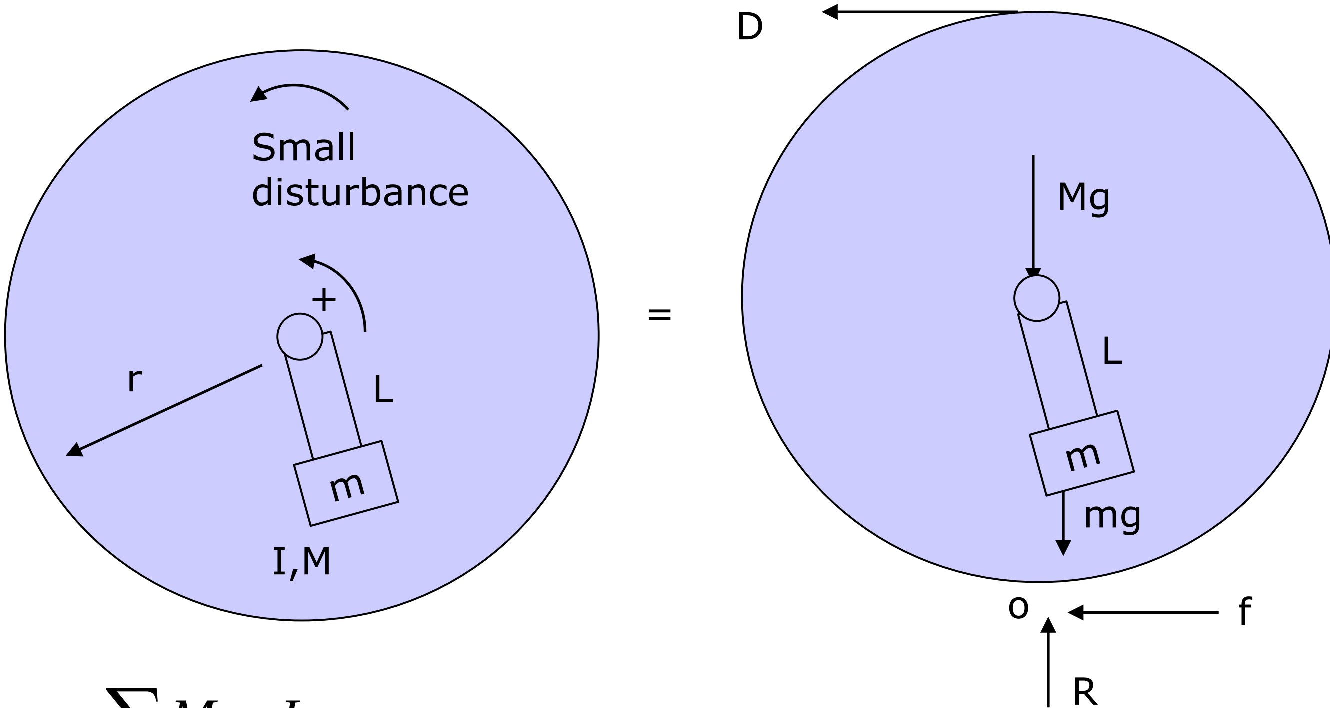
$$I^*\alpha = (L+r)D$$

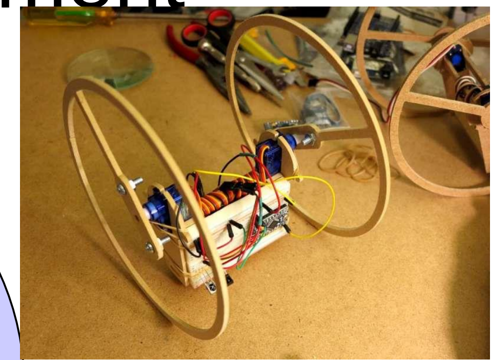
$$\frac{\Theta(s)}{D(s)} = \frac{(L+r)}{I^* s^2}$$

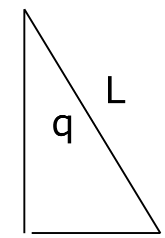
$$\frac{\Theta(s)}{\mathrm{T}(s)} < \frac{\Theta(s)}{D(s)}$$

Disturbances at L+r is bigger than Torque input

Self Balancing with Small Movement







$$\sum M = I\alpha$$

$$D2r - mgL\sin q = I^{*}\alpha$$

$$\therefore I^{*}\ddot{q} + mgLq = D2r$$

$$\therefore \frac{2r}{I^{*}s^{2} + \underline{mgL}} = \frac{Q(s)}{D(s)}$$

Small Movement → Linear Assumption

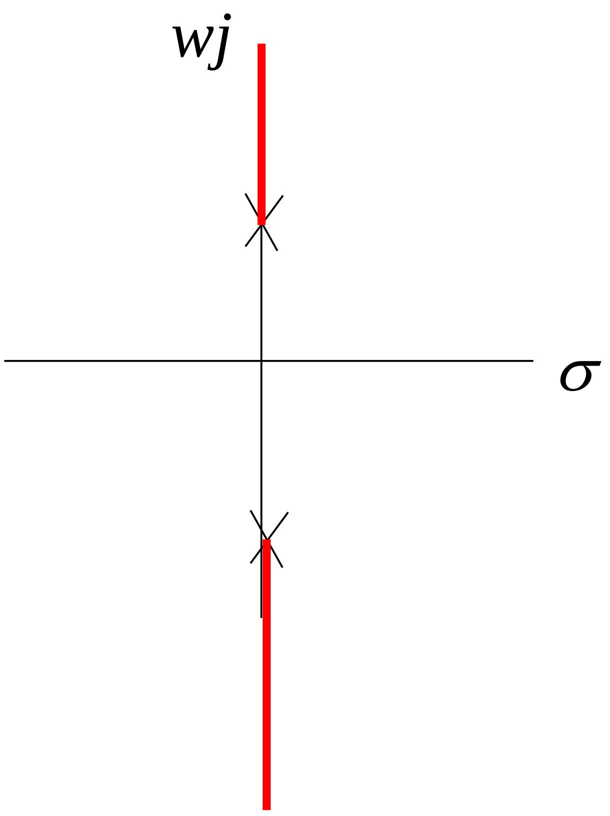
$$L\sin q = L(0 + q - \frac{1}{3!}q^{3} + ...) \text{ by Taylor series.}$$

$$if \;\; \text{q<<1}$$

$$L\sin q \approx Lq$$

Root Locus of Self Balancing

$$\frac{Q(s)}{D(s)} = \frac{2r}{I^{*}s^{2} + Lmg}$$



When w=0 (no velocity),
It is very stable

When w is increasing. (oscillation)
It is stable, too.

But, a Roly-Poly can fall with Strong force
What is wrong in our model?...

Think the assumption that sin q = q

If q >> 1, the assumption fails.

Self Balancing with Large Movement

$$\sum M = I\alpha$$

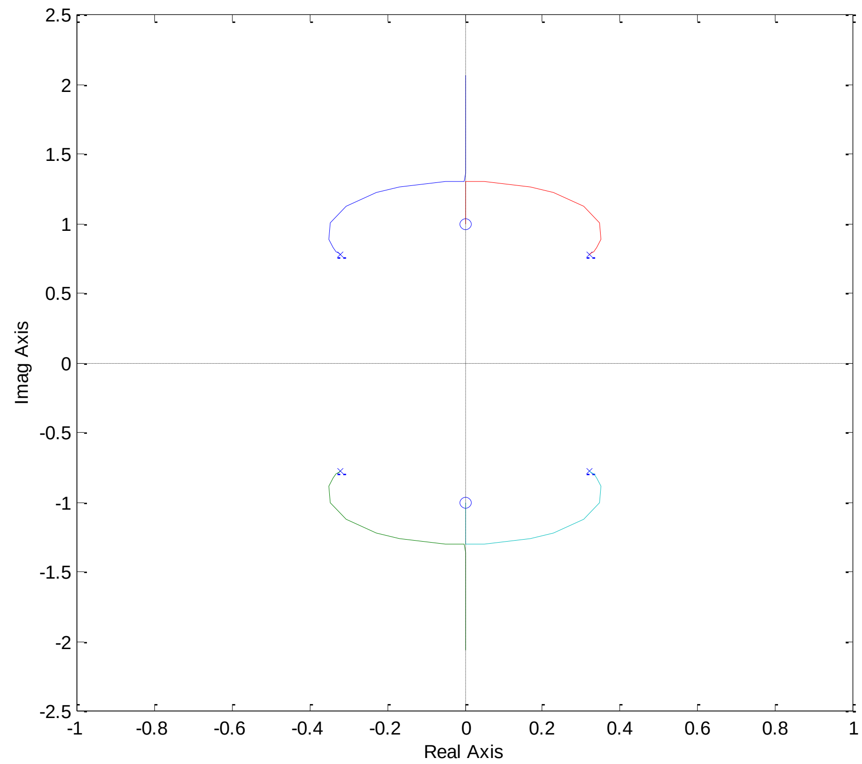
$$D2r - mgL\sin q = I^{*}\alpha$$

$$\therefore I^{*}\ddot{q} + mgL\sin q = D2r$$

$$I^{*}s^{2} + mg\frac{L}{s^{2}+1} = 2r\frac{\mathrm{Q}(s)}{\mathrm{D}(s)}$$

$$\therefore \frac{\mathrm{D}(s)}{\mathrm{Q}(s)} = \frac{s^{2}+1}{I^{*}s^{4} + I^{*}s^{2} + mgL}$$

If w increase, system becomes oscillatory.



rlocus(tf([1 0 1], [10 0 10 0 5]));

Self Balancing
with Small or Large Movement

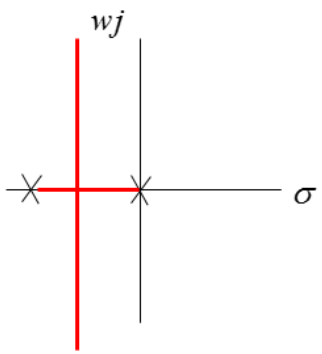
Small Movement =
Linear Assumption

$$\sum M = I\alpha$$

$$D2r - mgL\sin q = I^{*}\alpha$$

$$\therefore I^{*}\ddot{q} + mgLq = D2r$$

$$\therefore \frac{2r}{I^{*}s^{2} + mgLs} = \frac{\mathrm{Q}(s)}{\mathrm{D}(s)}$$



Always
Stable!

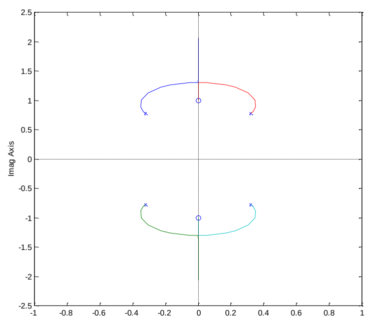
Large Movement
= Non linear Eq.

$$\sum M = I\alpha$$

$$D2r - mgL\sin q = I^{*}\alpha$$

$$\therefore I^{*}\ddot{q} + mgL\sin q = D2r$$

$$\therefore \frac{\mathrm{D}(s)}{\mathrm{Q}(s)} = \frac{s^{2}+1}{I^{*}s^{4} + I^{*}s^{2} + mgL}$$



Stability wrt
Inputs