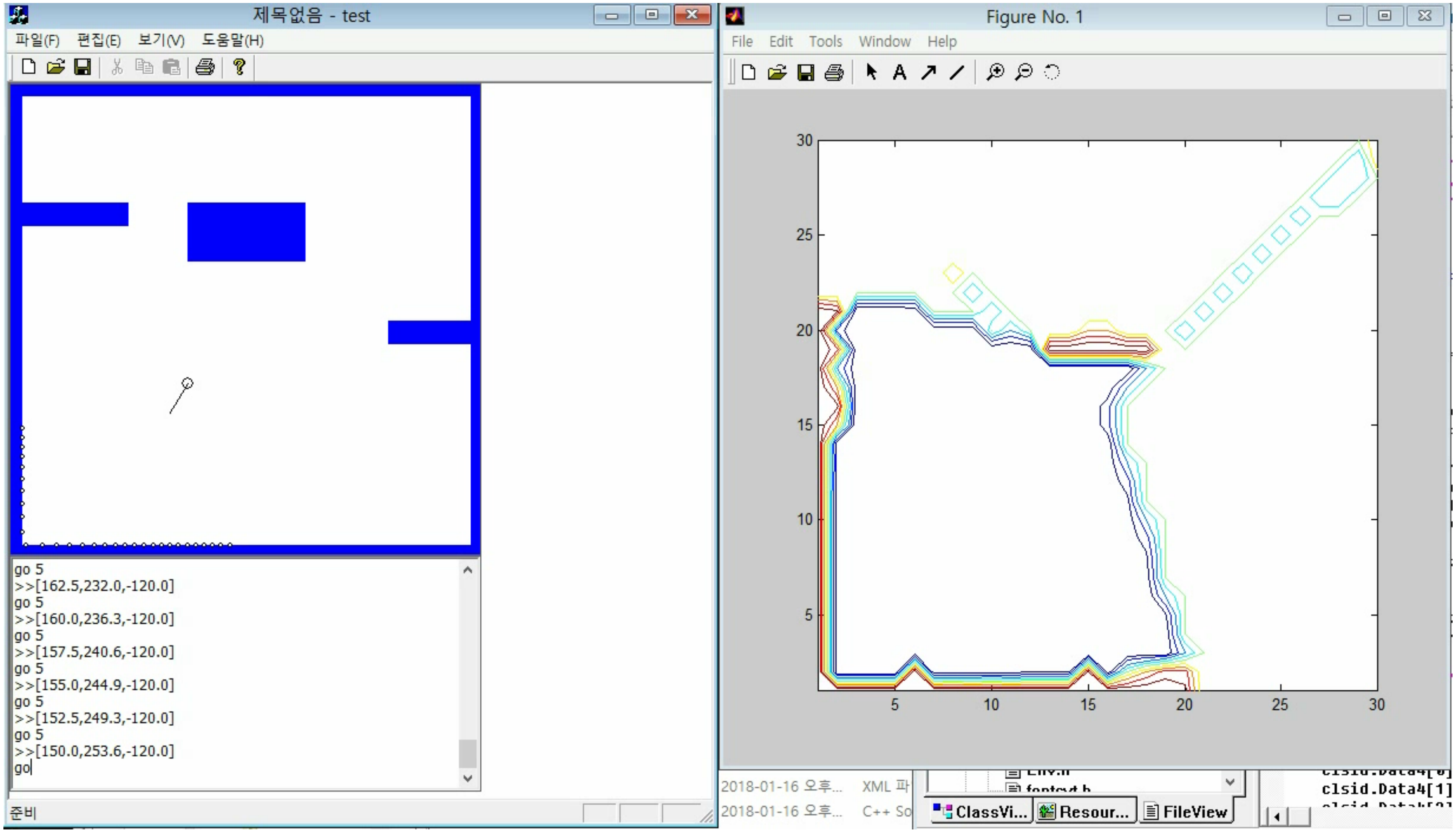
Probabilistic Robotics
Map Building

양정연

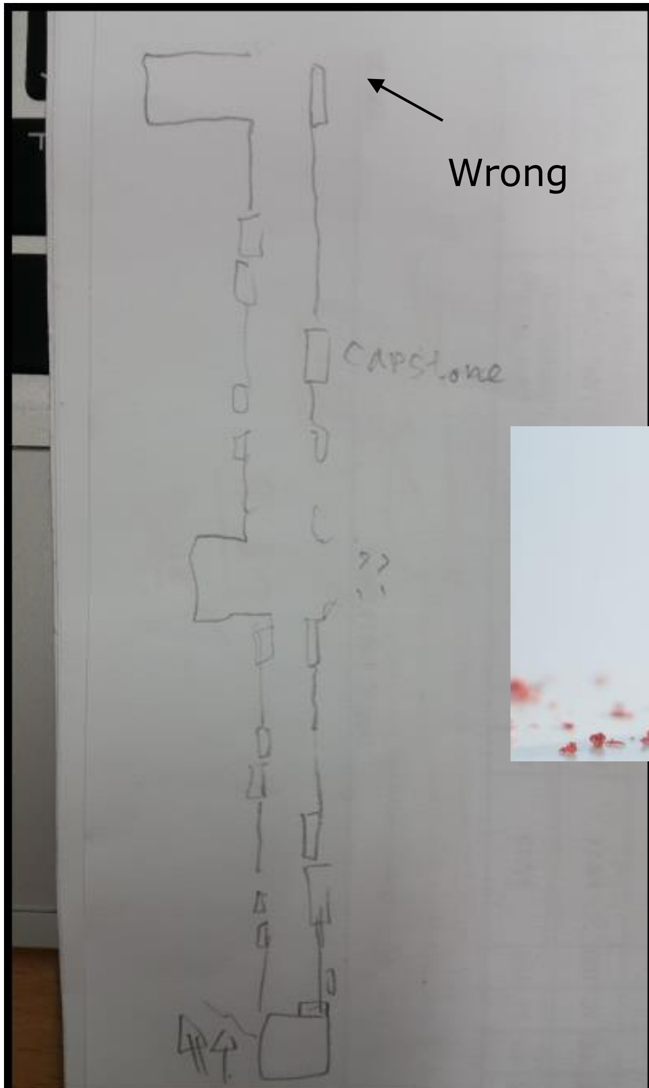
2020/12/10

What is a Map or Mapping



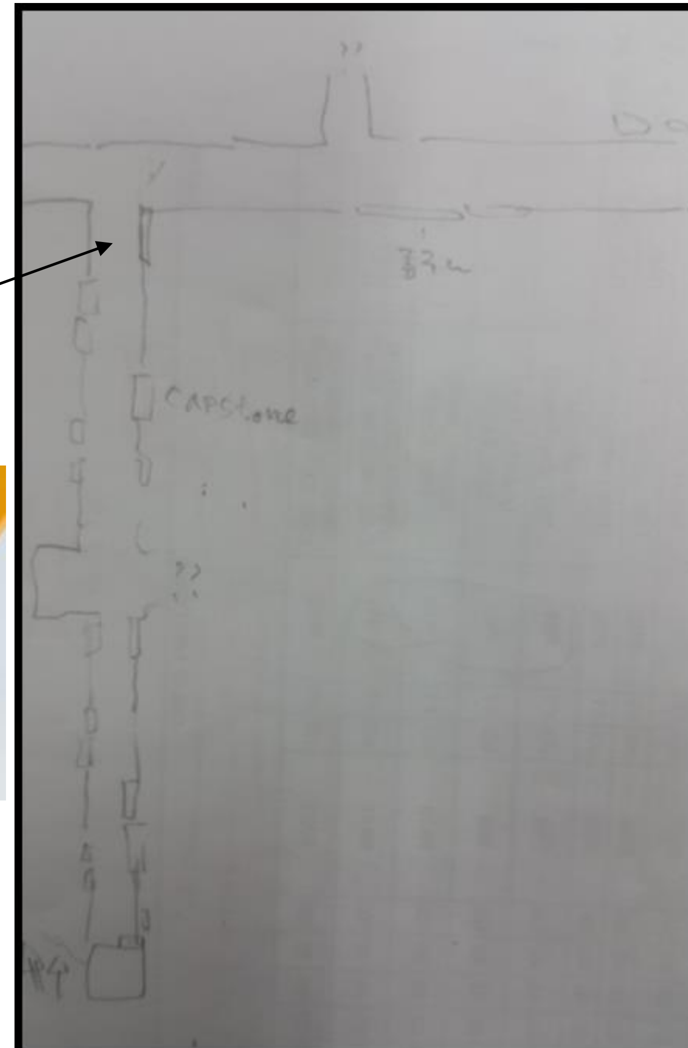
Introduction: Manual Mapping

Map Drawings with Pen and **Eraser**



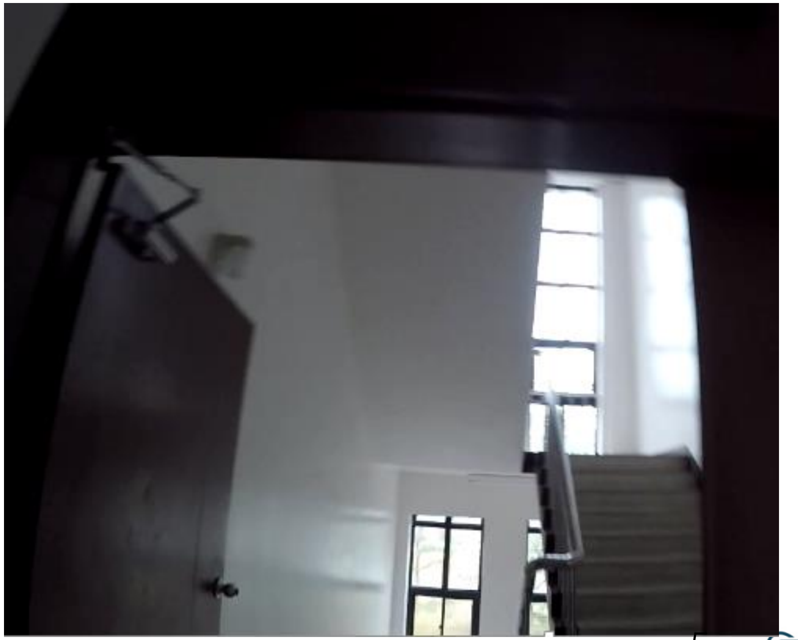
Wrong

Fix it



Problems of Map Drawing

- 1. Distance is NOT Clear with a Video.
- 2. Wrong Position should be Fixed later
- 3. “Stair” is unclear
- 4. Missing Area
- 5. Noisy Area



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Engineering Ways: Map has these Problems

- 1. Distance Problem
  - Distance measurement (or Distance metric) is possible with a laser scanner or Kinect-like point cloud devices
- 2. Update Map (Unclear, Missing and Noisy area)
  - With a Single video, we missed most of map information
  - Thus, **map update is very essential (like pencil with eraser)**
- 3. Where am I?
  - In spite of all, The current position information is missing.

Goal of SLAM is,

$$p(x_{0:t}, m \mid z_{1:t}, u_{1:t})$$

position Map Observation Control input

- SLAM: Simultaneous Localization and Mapping
  - It is NOT Easy doing localization and mapping at the same time
  - Localization requires Map
  - Mapping requires Localization

  - It is an egg-and-hen problem

Rao-Blackwellization

- Doing Factorization

$$p(x_{0:t}, m \mid z_{1:t}, u_{1:t})$$

Rao-Blackwell-Kolmogorov Theorem

$$p(m \mid x_{1:t}, z_{1:t})\, p(x_{0:t} \mid z_{1:t}, u_{1:t})$$

By Murphy in 1999

Mapping

Localization

SLAM

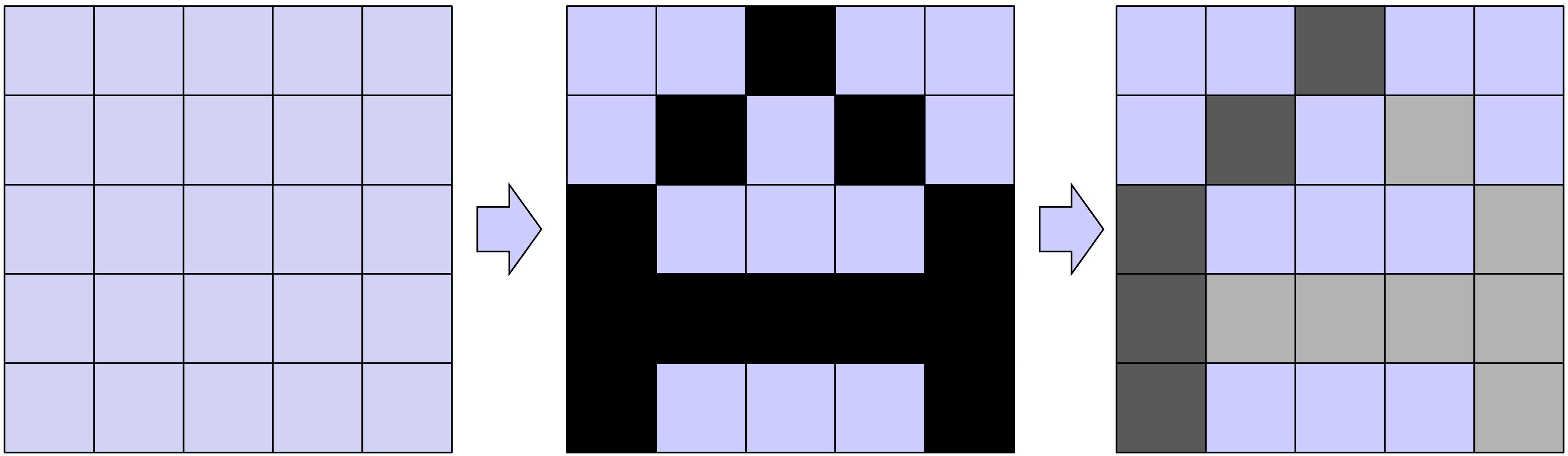
Mapping from Rao-Blackwellization

$$p(m \mid x_{1:t}, z_{1:t})\, p(x_{0:t} \mid z_{1:t}, u_{1:t})$$

- Mapping
  - Assumption
  - **If we know X**, then observation Z with X can generate a Map
- Then, How to generate a map?
  - The Easiest one is using a GRID map

    → Occupancy Grid Mapping

Occupancy Grid Mapping



Empty space

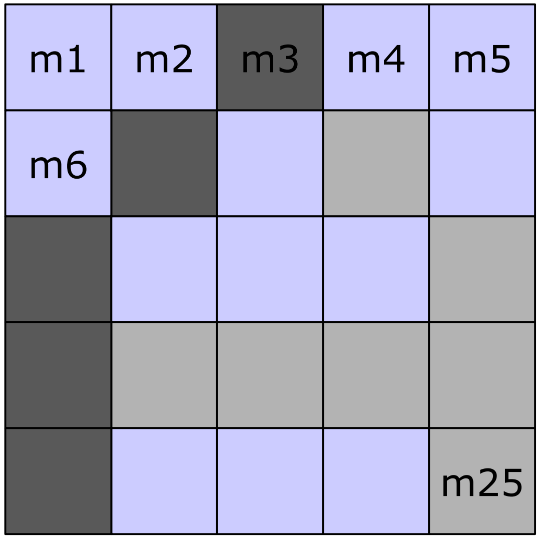
0 for empty
1 for occupancy

Probabilistic way
0< Prob. of occupancy<1

- Prob. Of occupancy :

$$p(m_i) = \begin{cases} 0 : \text{empty} \\ 1 : occupied \\ otherwise, \end{cases}$$

Each Grid is Independent



$$p(m) = p(m_1)\,p(m_2)\ldots p(m_{25})$$

$$p(m) = \prod_{i=1}^{25} p(m_i)$$

Probability is always not greater than 1.
Thus, multiplication becomes very smaller.

→ Logarithmic operation must be used.

$$\log p(m) = \log \prod_{i=1}^{25} p(m_i)$$

$$= \sum_i \log(p(m_i))$$

$$RB:\ p(m \mid x_{1:t}, z_{1:t})\, p(x_{0:t} \mid z_{1:t}, u_{1:t})$$

$$p(m \mid x_{1:t}, z_{1:t})$$

$$\text{from } p(m_i \mid x_{1:t}, z_{1:t})$$

Calculate P(m) from p(mi)

Probability of a Cell Occupancy

$$p(m_i \mid x_{1:t}, z_{1:t}) \rightarrow p(m \mid x_{1:t}, z_{1:t})$$

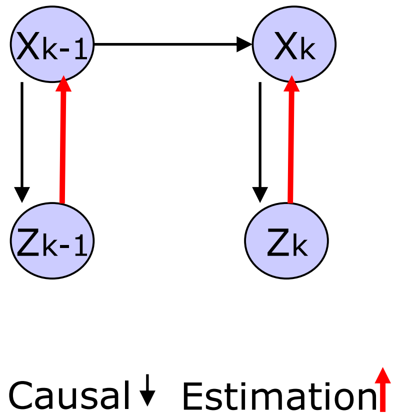
- Remind Bayesian Rule

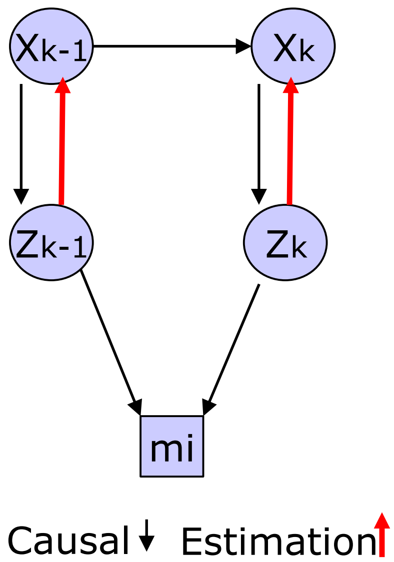
$$p(A,B \mid C) = \frac{p(A,B,C)}{p(C)} = \frac{p(A,B,C)}{p(B,C)} \frac{p(B,C)}{p(C)} = p(A \mid B,C)P(B \mid C)$$

$$\frac{p(A,B \mid C)}{P(B \mid C)} = p(A \mid B,C)$$

$$p(m_i \mid x_{1:t}, z_{1:t}) = p(m_i \mid x_{1:t}, z_{1:t-1}, z_t)$$

Remind that Map is update through the kth step





$$\frac{p(A,B|C)}{P(B|C)} = p(A|B,C)$$

$$p(m_i \mid x_{1:t}, z_{1:t}) = p(m_i \mid x_{1:t}, z_{1:t-1}, z_t)$$

$$= \frac{p(m_i, x_{1:t}, z_{1:t-1}, z_t)}{p(x_{1:t}, z_{1:t-1}, z_t)} = \frac{p(z_t, m_i, x_{1:t}, z_{1:t-1})}{p(z_t, x_{1:t}, z_{1:t-1})} = \frac{p(z_t, m_i, x_{1:t}, z_{1:t-1})}{p(m_i, x_{1:t}, z_{1:t-1})} \frac{p(m_i, x_{1:t}, z_{1:t-1})}{p(z_t, x_{1:t}, z_{1:t-1})}$$

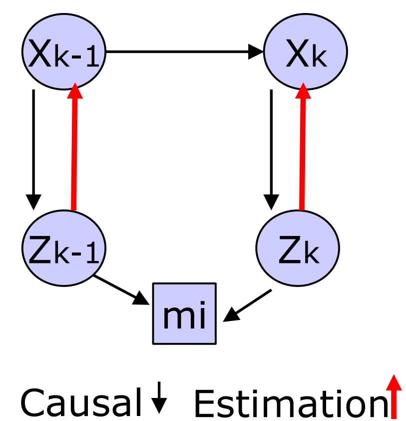
$$= p(z_t \mid m_i, x_{1:t}, z_{1:t-1}) \frac{p(m_i, x_{1:t}, z_{1:t-1})}{p(x_{1:t}, z_{1:t-1})} \frac{p(x_{1:t}, z_{1:t-1})}{p(z_t, x_{1:t}, z_{1:t-1})}$$

$$= p(z_t \mid m_i, x_t, \cancel{z_{1:t-1}})\, \mathrm{p}(\mathrm{m}_i \mid x_{1:t-1}, z_{1:t-1}) \frac{1}{p(z_t \mid x_{1:t}, z_{1:t-1})}$$

$$= p(z_t \mid m_i, x_t)\, \mathrm{p}(\mathrm{m}_i \mid x_{1:t-1}, z_{1:t-1}) \frac{1}{p(z_t \mid x_{1:t}, z_{1:t-1})}$$

$$= \frac{p(z_t, m_i, x_t)}{p(m_i, x_t)} \mathrm{p}(\mathrm{m}_i \mid x_{1:t-1}, z_{1:t-1}) \frac{1}{p(z_t \mid x_{1:t}, z_{1:t-1})}$$

$$= \frac{p(m_i, z_t, x_t)}{p(z_t, x_t)} \frac{p(z_t, x_t)}{p(x_t)} \frac{p(x_t)}{p(m_i, x_t)} \mathrm{p}(\mathrm{m}_i \mid x_{1:t-1}, z_{1:t-1}) \frac{1}{p(z_t \mid x_{1:t}, z_{1:t-1})}$$

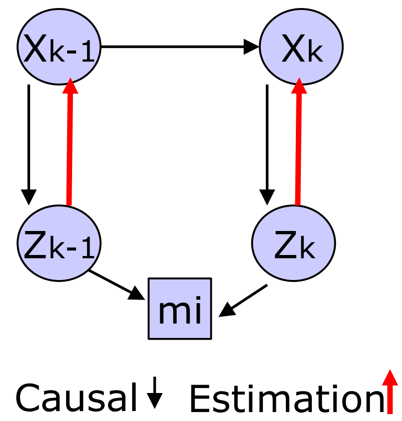


$$p(m_i \mid x_{1:t}, z_{1:t}) = p(m_i \mid x_{1:t}, z_{1:t-1}, z_t)$$

$$= \frac{p(m_i, z_t, x_t)}{p(z_t, x_t)} \frac{p(z_t, x_t)}{p(x_t)} \frac{p(x_t)}{p(m_i, x_t)} p(m_i \mid x_{1:t-1}, z_{1:t-1}) \frac{1}{p(z_t \mid x_{1:t}, z_{1:t-1})}$$

$$= p(m_i \mid z_t, x_t) \frac{p(z_t \mid x_t)}{p(m_i \mid x_t)} \frac{p(m_i \mid x_{1:t-1}, z_{1:t-1})}{p(z_t \mid x_{1:t}, z_{1:t-1})}$$

$$= p(m_i \mid z_t, x_t) \frac{p(z_t \mid x_t)}{p(m_i)} \frac{p(m_i \mid x_{1:t-1}, z_{1:t-1})}{p(z_t \mid x_{1:t}, z_{1:t-1})}$$



Map mi is associated with the combination of x and z,
But not with x.

$$p(m_i \mid x_{1:t}, z_{1:t}) = p(m_i \mid x_t, z_t) \frac{p(z_t \mid x_t)}{p(m_i)} \frac{p(m_i \mid x_{1:t-1}, z_{1:t-1})}{p(z_t \mid x_{1:t}, z_{1:t-1})}$$

Binary Attribute of "Occupancy or Empty" → Simplify the Equations

- Occupancy

$$p(m_i \mid x_{1:t}, z_{1:t}) = p(m_i \mid x_t, z_t) \frac{p(z_t \mid x_t)}{p(m_i)} \frac{p(m_i \mid x_{1:t-1}, z_{1:t-1})}{p(z_t \mid x_{1:t}, z_{1:t-1})}$$

- Empty

$$p(\neg m_i \mid x_{1:t}, z_{1:t}) = p(\neg m_i \mid x_t, z_t) \frac{p(z_t \mid x_t)}{p(\neg m_i)} \frac{p(\neg m_i \mid x_{1:t-1}, z_{1:t-1})}{p(z_t \mid x_{1:t}, z_{1:t-1})}, \quad \text{"}\neg = Not\text{"}$$

- We do NOT calculate "BLUE" probability

$$\frac{p(m_i \mid x_{1:t}, z_{1:t})\, p(m_i)}{p(m_i \mid x_t, z_t)\, p(m_i \mid x_{1:t-1}, z_{1:t-1})} = \frac{p(z_t \mid x_t)}{p(z_t \mid x_{1:t}, z_{1:t-1})}$$

$$= \frac{p(\neg m_i \mid x_{1:t}, z_{1:t})\, p(\neg m_i)}{p(\neg m_i \mid x_t, z_t)\, p(\neg m_i \mid x_{1:t-1}, z_{1:t-1})}$$

Log Odds Notation

$$\frac{p(m_i \mid x_{1:t}, z_{1:t})\, p(m_i)}{p(m_i \mid x_t, z_t)\, p(m_i \mid x_{1:t-1}, z_{1:t-1})} = \frac{p(\neg m_i \mid x_{1:t}, z_{1:t})\, p(\neg m_i)}{p(\neg m_i \mid x_t, z_t)\, p(\neg m_i \mid x_{1:t-1}, z_{1:t-1})}$$

$$\frac{p(m_i \mid x_{1:t}, z_{1:t})}{p(\neg m_i \mid x_{1:t}, z_{1:t})} = \frac{p(m_i \mid x_t, z_t)\, p(m_i \mid x_{1:t-1}, z_{1:t-1})\, p(\neg m_i)}{p(\neg m_i \mid x_t, z_t)\, p(\neg m_i \mid x_{1:t-1}, z_{1:t-1})\, p(m_i)}$$

$$\therefore \frac{p(m_i \mid x_{1:t}, z_{1:t})}{1 - p(m_i \mid x_{1:t}, z_{1:t})} = \frac{p(m_i \mid x_t, z_t)}{1 - p(m_i \mid x_t, z_t)} \times \frac{1 - p(m_i)}{p(m_i)} \times \frac{p(m_i \mid x_{1:t-1}, z_{1:t-1})}{1 - p(m_i \mid x_{1:t-1}, z_{1:t-1})}$$

$\log\ odds\ representation$ $\quad \log\ odds : l(a) \triangleq \log \frac{p(a)}{1 - p(a)}$

$$l(m_i \mid x_{1:t}, z_{1:t}) = \log\left(\frac{p(m_i \mid x_{1:t}, z_{1:t})}{p(\neg m_i \mid x_{1:t}, z_{1:t})} \right) = \log\left(\frac{p(m_i \mid x_{1:t}, z_{1:t})}{1 - p(m_i \mid x_{1:t}, z_{1:t})} \right)$$

$$l(m_i \mid x_{1:t}, z_{1:t}) = l(m_i \mid x_t, z_t) - l(m_i) + l\left(m_i \mid x_{1:t-1}, z_{1:t-1}\right)$$

Finally, Grid Map Probability

$$\log\ \text{Odds}: \ l(a) \triangleq \log \frac{p(a)}{1-p(a)}$$

$$\frac{p(a)}{1-p(a)} = e^{l(a)}$$

$$p(a)(1+e^{l(a)}) = e^{l(a)}$$

$$\therefore\ p(a) = \frac{e^{l(a)}}{1+e^{l(a)}} = 1 - \frac{1}{1+e^{l(a)}}$$

$$l(m_i \mid x_{1:t}, z_{1:t}) = l(m_i \mid x_t, z_t) + l\left(m_i \mid x_{1:t-1}, z_{1:t-1}\right) - l(m_i)$$

$$\therefore\ p(m_i \mid x_{1:t}, z_{1:t}) = 1 - \frac{1}{1+\exp\left[l(m_i \mid x_{1:t}, z_{1:t})\right]}$$

Summary

- Goal of SLAM $p(x_{0:t}, m \mid z_{1:t}, u_{1:t})$
- Rao-Blackwellization $p(m \mid x_{1:t}, z_{1:t})\, p(x_{0:t} \mid z_{1:t}, u_{1:t})$
- Map building $p(m \mid x_{1:t}, z_{1:t})$
- Map has binary attribute : occupancy grid map

$$p(m \mid x_{1:t}, z_{1:t}) \text{ or } p(\neg m \mid x_{1:t}, z_{1:t})$$

- Map probability $p(m_i \mid x_{1:t}, z_{1:t}) = 1 - \dfrac{1}{1 + \exp\left[l(m_i \mid x_{1:t}, z_{1:t})\right]}$

What is the Physical Meaning of Occupancy grid map?

$$\text{log Odds}: \; l(a) \triangleq \log \frac{p(a)}{1-p(a)}$$

At t>0 $\quad p(m_i \mid x_{1:t}, z_{1:t}) = 0 \quad p(m_i \mid x_{1:t}, z_{1:t}) = 1 \quad p(m_i \mid x_{1:t}, z_{1:t}) = ?$

At t=0 $\quad p(m_i) = 0 \quad p(m_i) = 1 \quad p(m_i) = ?$

- P(m)=0 means, “I am sure that it is an Empty”
- Thus, when we start mapping, **the initial prob. = 0.5**

$$l_0(\mathrm{m}) = \log \frac{p(\mathrm{m})}{1-p(\mathrm{m})} = 0 \quad \therefore p(m) = 0.5$$

Map Update Strategy $p(m \mid x_{1:t}, z_{1:t})$ with Inverse measurement(or Sensor) model

$$l(m_i \mid x_{1:t}, z_{1:t}) = l(m_i \mid x_t, z_t) + l\left(m_i \mid x_{1:t-1}, z_{1:t-1}\right) - l(m_i)$$

$$l_t = l(m_i \mid x_t, z_t) + \mathrm{l}_{t-1} - l_0\left(m_i\right)$$

$$= l(m_i \mid x_t, z_t) + \mathrm{l}_{t-1} - l_0$$

$$t > 0 : p(m_i \mid x_{1:t}, z_{1:t}) \rightarrow l_t$$

$$t = 0 : p(m_i) \rightarrow l_0$$

- If we calculate $l(m_i \mid x_t, z_t)$, then we update a map

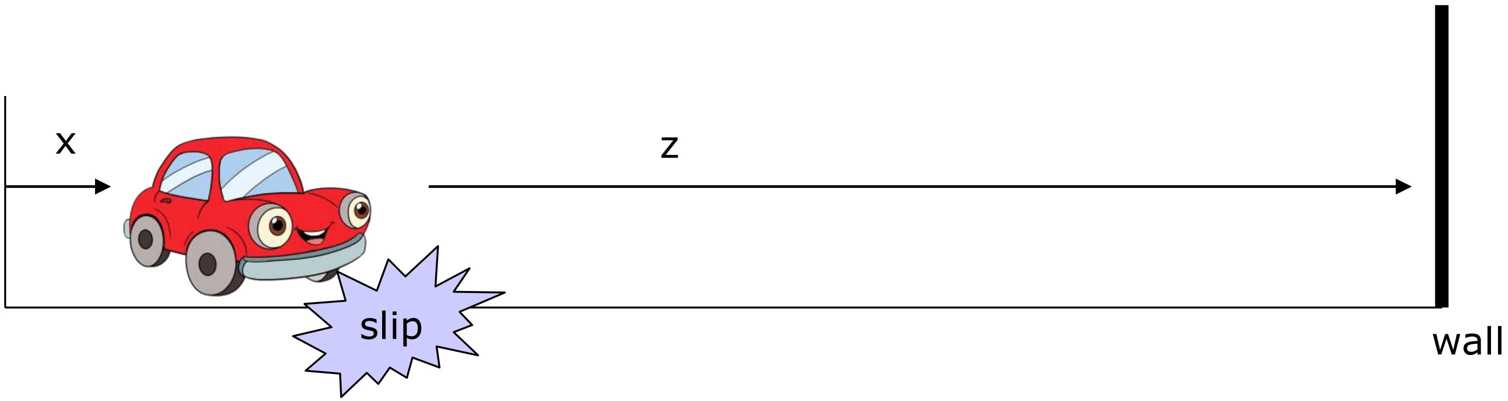
- It is called

  $l(m_i \mid x_t, z_t) \rightarrow p(m_i \mid x_t, z_t) = \text{Inverse measurement model}$

  – From the current position, x and the current measurement, z, we estimate prob. Of whether a map mi is empty or occupied.

  → It is different with a classification probability.

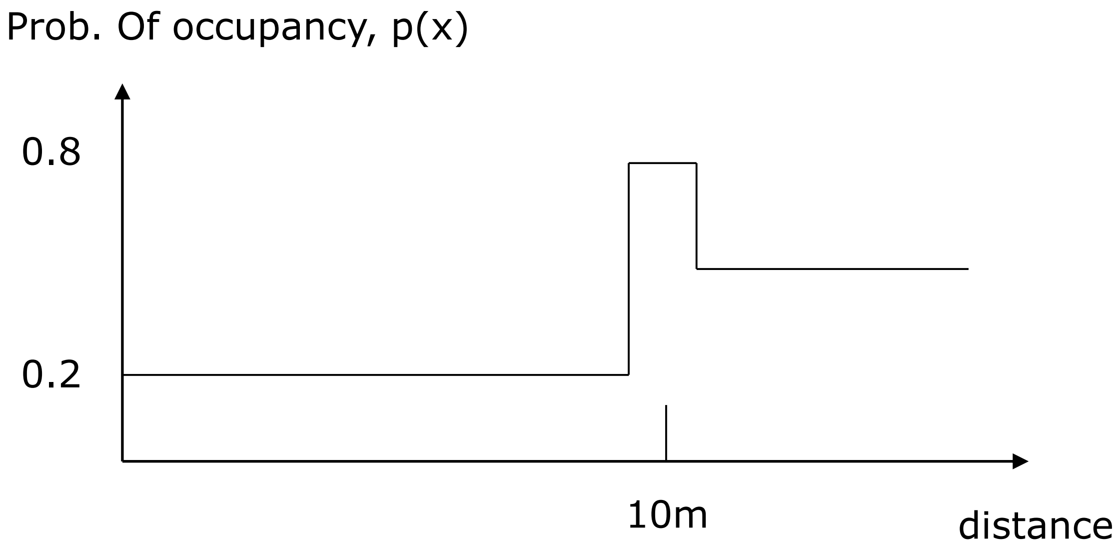
Why Inverse Sensor Model is required?



Our measurement, z has noise

Inverse Sensor Model

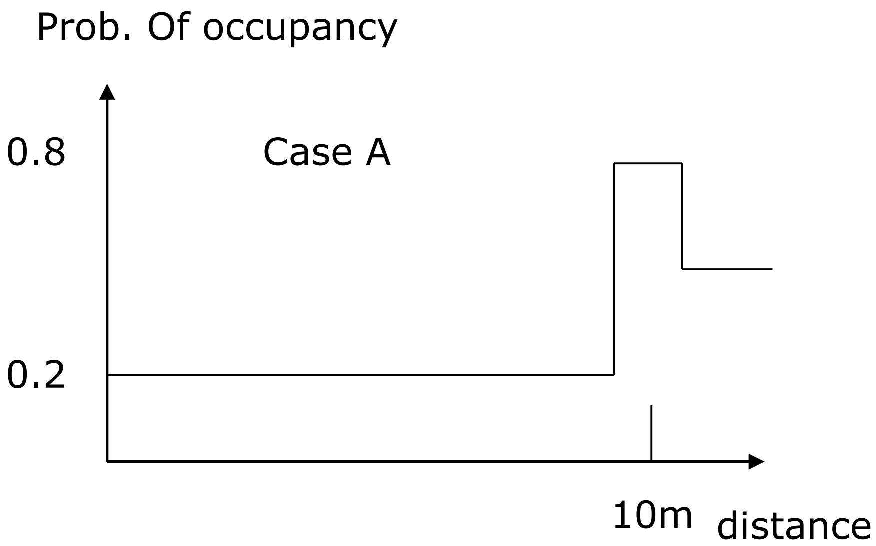
Measurement:
Distance is 10m

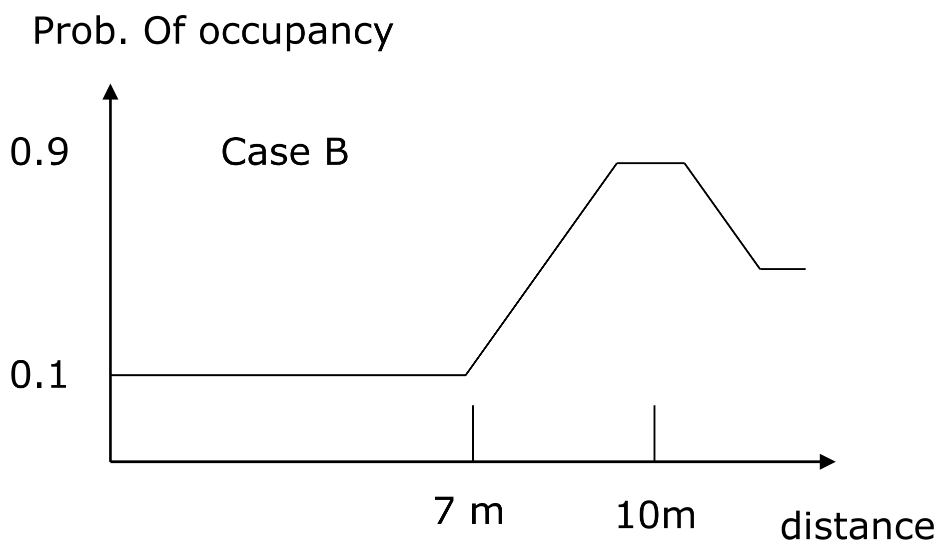


- Prob. of occupancy z=10, prob(m|x,z) = 0.8
- Prob. Of occupancy z=5, prob(m|x,z) = 0.2

- Inverse sensor model is dependent of Sensor itself

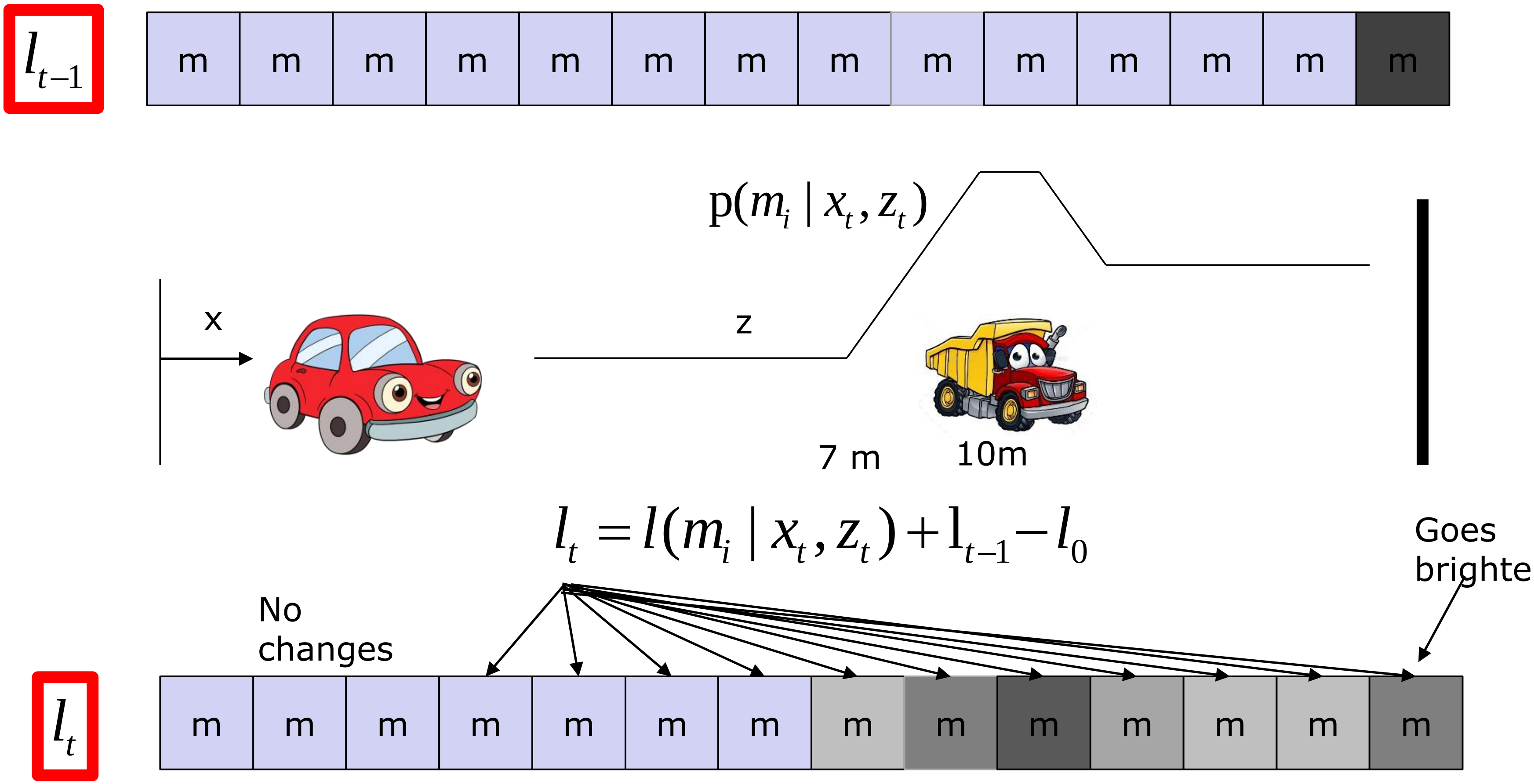
Inverse Sensor Model : Sensor Performance



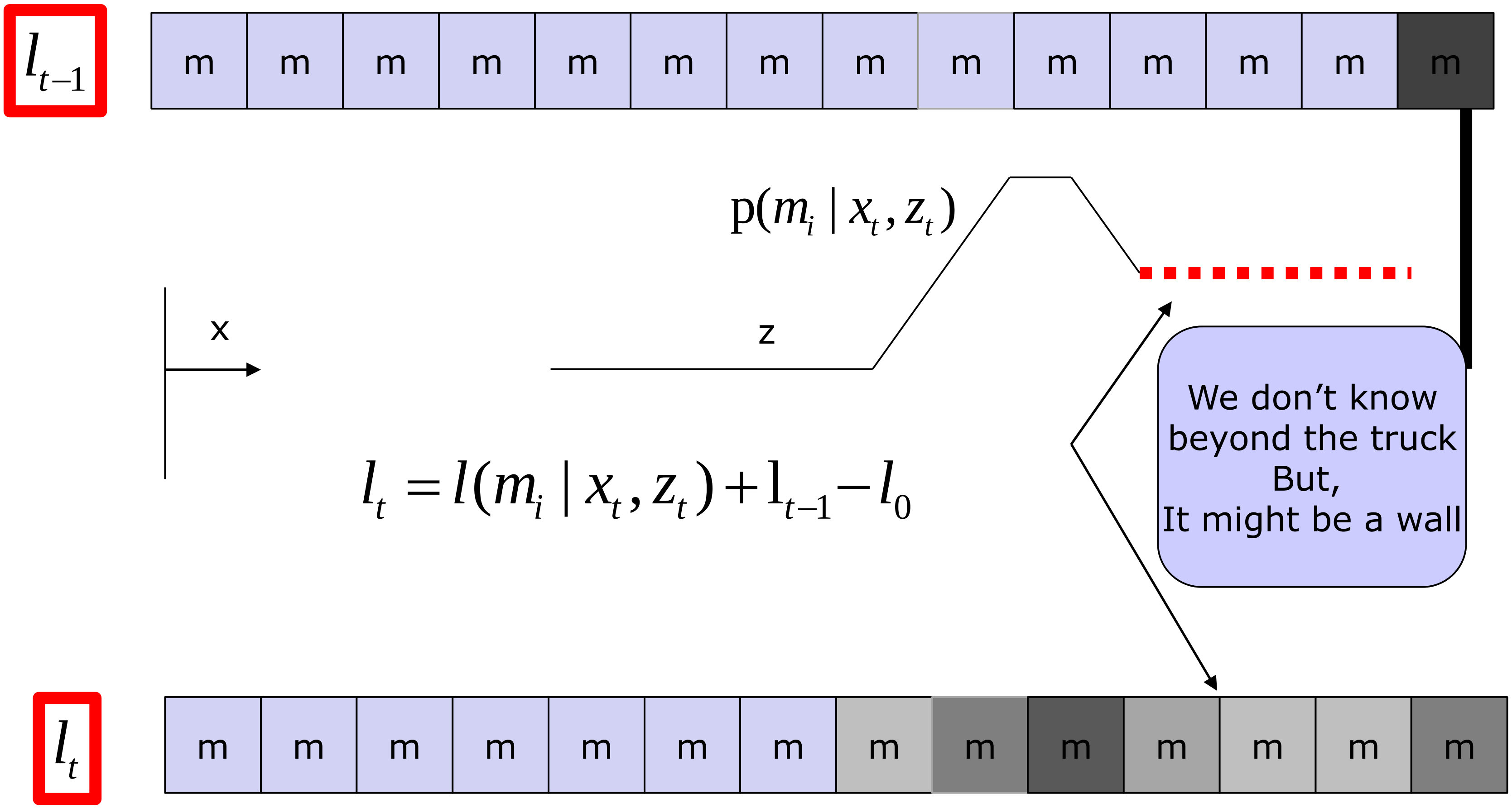


- Which one is better?
- Case B has poor performance over 7m.
  - The sensor does not determined whether it is occupied or not over 7m

Why Inverse S.M. is so Important? And is Not a Recognition Rate?



What is the meanings of Inverse S.M. beyond an obstacle?

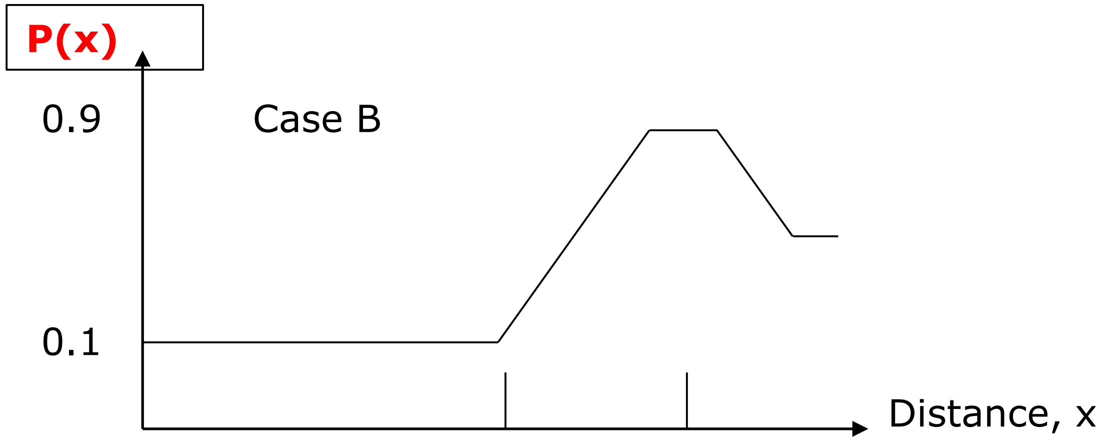


$$l_t = l(m_i \mid x_t, z_t) + l_{t-1} - l_0$$

Inverse S.M. is different with Recognition Rate

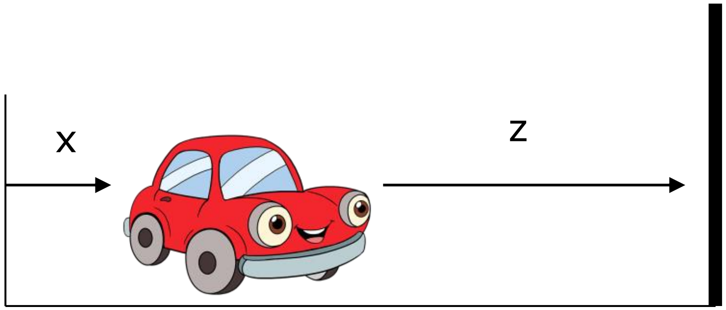
- Recognition rate tells us,
  - It has a probability of whether it is true or not

- Inverse Sensor(or Measurement) model tells us,
  - NO interest about whether an obstacle is or not
  - I am very interested in ***WHERE an obstacle is now***?
  - Probability of distance is the major interest. → P = P(x)



Simulation: Test.m

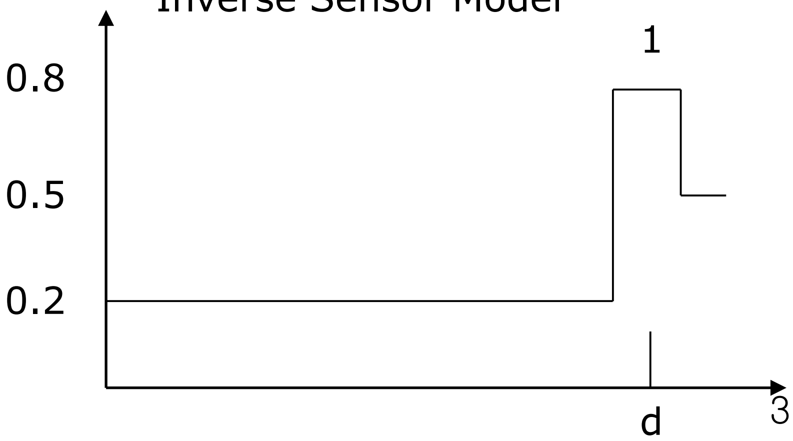
```
% measurement
z = 10-x;
z = z+randn;

% map update
xi = floor(x*n/10);
p= 0;
for j=1:n
   mi = j;

   if (mi>xi)
      if (mi<(z-0.5)*n/10)
         p = Pe;
      else
         if (mi>(z+0.5)*n/10)
            p=Pp;
         else
            p=Po;
         end
      end

      lm(j) = lm(j)+ log(p/(1-p));
   end
end
```



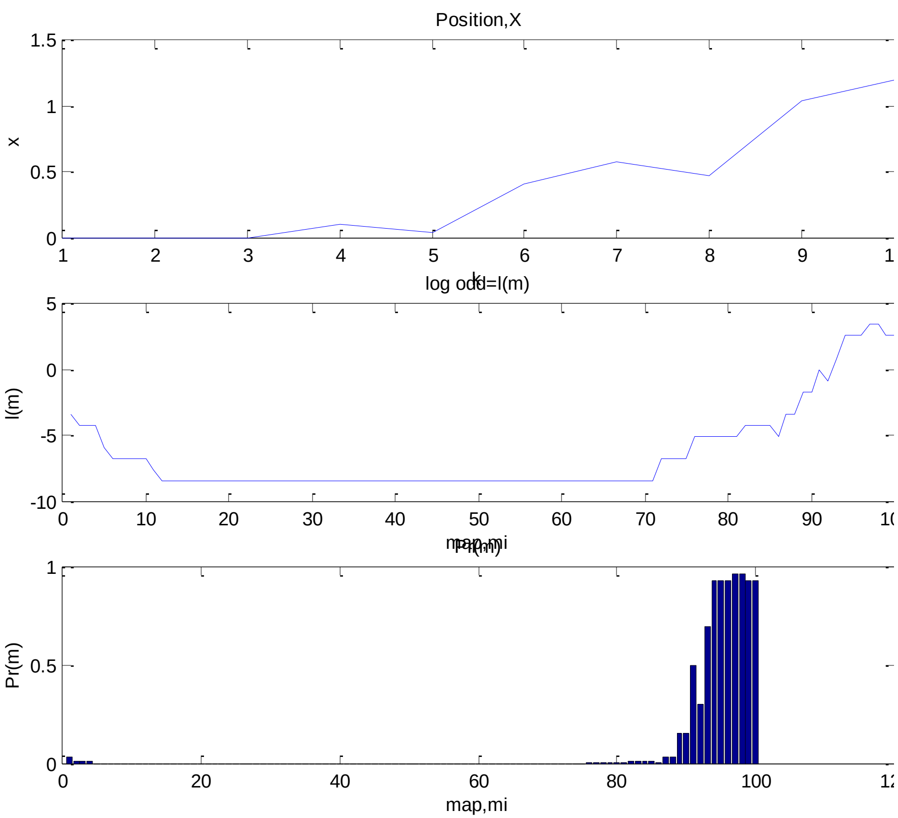
$$x_{k+1} = x_k + 0.5N(0,1)$$

$$z = 10 - x + N(0,1)$$

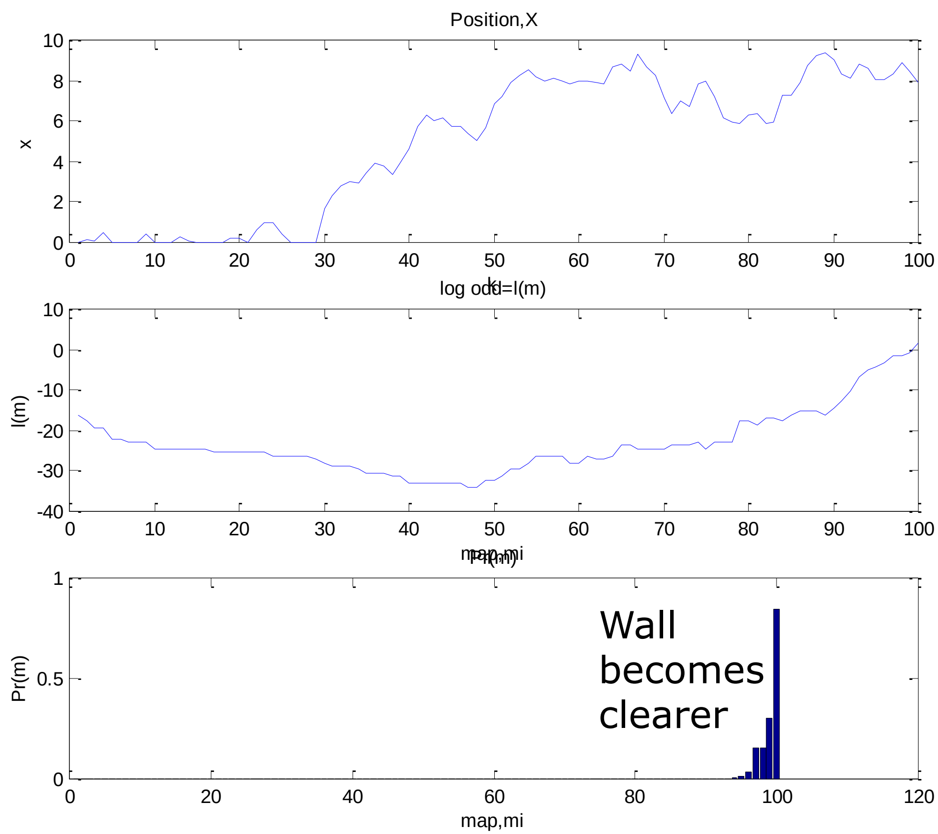
Tough Noise!



Simulation Result



t=0 to 10



t=0 to 100

Extends from 1D to 2D
What will be required?